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Eliminating between the equation of the torus and that of the plane, we obtain $\left(x^2+y^2+\frac{r^2}{p^2}x\right)^2 - 2q^2(x^2+y^2) + 2r^2x^2 = -p^4$, which by expansion and reduction becomes

$$(R^2x^2+p^2y^2)^2 - 2R^2p^4x^2 - 2p^4q^2y^2 = -p^8.$$

This is the equation of an oblique projection of the intersection .To obtain the curve in its own plane, we must substitute for x, x' cos (angle between tangent plane and plane $x\ y$) $= (p \div R)x'$. Therefore

$$p^4(x'^2+y^2)^2 - 2p^6x'^2 - 2p^4q^2y^2 = -p^8,$$

or, dividing by p^4 and dropping primes,

$$(x^2+y^2)^2 - 2p^2x^2 - 2q^2y^2 = -p^4.$$

Add to both sides of this equation $2p^2(x^2+y^2)$, then

$$(x^2+y^2)^2 - 2p^2x^2 - 2q^2y^2 + 2p^2x + 2p^2y^2 = 2p^2(x^2+y^2) - p^4$$

or

$$(x^2+y^2)^2 - 2y^2(q^2-p^2) = 2p^2(x^2+y^2) - p^4$$

$$(x^2+y^2)^2 - 2y^2(R^2+r^2-R^2+r^2) = 2p^2(x^2+y^2) - p^4$$

$$(x^2+y^2)^2 - 2p^2(x^2+y^2) + p^4 = 4r^2y^2.$$

$$\therefore \quad x^2+y^2-p^2 = \pm 2ry;$$

$$\therefore \quad x^2+y^2 \pm 2ry + r^2 = R^2,$$

and

$$x^2+(y\pm r)^2 = R^2.$$

Hence the section is two circles whose radius is R, and whose centers are at a distance from the center of the torus equal to the radius of the generating circle of the torus.

In a similar manner we find that when the generating curve is an ellipse, the section is two ellipses whose semi axes are R and $\sqrt{(R^2-a^2+b^2)}$, a and b being the semi axes of the generating ellipse. So with a hyperbolic generatrix, we obtain hyperbolas whose semi axes are R and $\sqrt{(R^2-a^2-b^2)}$. In this case we must have $a > R$ in order to render the problem possible. With parabolic generatrix we have a parabola whose parameter (latus rectum) is $4(R+p)$,* that of the generatrix being $4p$. With the conjugate hyperbola for generatrix the section is *imaginary*, the equation of its projection being $R^2x^2 + (R^2+a^2)(y \mp a\sqrt{-1})^2 = R^2(R^2-a^2)$.

DIFFERENTIATION.

BY J. B. MOTT, NEOSHO, MISSOURI.

HAVING seen several explanations of the method of finding the first differential coefficient, I offer the following:

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*In this case R is the distance from the center of the torus to the vertex of the generating parabola.

In the equation $u = ax^2$ (1), let $x = x + n$ and suppose from the start that the value of n is infinitely small. There will then be no finite change in the new equation of the values of either u or x. We shall have

$$u = a(x + n)^2 = ax^2 + 2anx + an^2 \ldots\ldots\ldots (2)$$

Subtracting (1) from (2) gives $u - u = 2anx + an^{2*}$ or $\frac{u - u}{n} = 2ax + an$.

Now as $n = 0$, so far as any thing finite is concerned, therefore

$$\frac{u - u}{n} = \frac{0}{0} = 2ax \ldots\ldots\ldots\ldots\ldots (3)$$

In the Calculus to preserve some traces of u and x from the fraction $(u - u) \div n = 0 \div 0$, authors generally put du for $u - u$ and dx for n, d being used merely as an abreviation for the word differential, the meaning of which I understand to be infinitely small. According to this notation, from (3) we have $\frac{u - u}{n} = \frac{0}{0} = \frac{du}{dx} = 2ax$.

In the same way the following equations; $u = ax^3$, $u = ax^2 + bx^3$, $u = ax + bx^2 + cx^3$, $u = a^2x - 4ax^2 + 4x^3$, &c., may be differentiated or transformed, respectively, to $\frac{0}{0} = \frac{du}{dx} = 3ax^2$, $2ax + 3bx^2$, $a + 2bx + 3cx^2$, $a^2 - 8ax + 12x^2$, &c.

The right hand members in the new equations are obtained most readily by multiplying each term of the primative by the exponent of x and diminishing the exponent by unity.

The fraction $\frac{du}{dx} = \frac{0}{0}$ may come from any quantity, thus $N = \frac{N}{1} = \frac{N}{1} \times \frac{0}{0} = \frac{0}{0}$, where N may be any whole number or fraction or 0 or ∞; but there are particular values to this fraction according to the conditions of the problem or equation from which it is derived.

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*If by the supposition that n is infinitely small it is meant that $n = 0$, then

$$u - u' = 2anx + an^2 = 0,$$

by reason of the zero factor in $2anx$ and an^2, and no logical inference can be drawn from the equation. But if n is supposed to be some finite quantity, then

$$\frac{u - u'}{n} = 2ax + an$$

is an intelligible equation; and it is easily seen that the ratio of $u - u'$ to n approaches to $2ax$ as n approaches to zero; from which we infer that the *limit* of the *ratio* of $u - u'$ to n is $2ax$. Therefore "When variable quantities become infinitely great, or infinitely small, their *limiting ratio* may frequently be determined, though the quantities themselves in such state elude our comprehension."—Ed.

From the foregoing I infer that, by substituting as above, any equation containing two variables, of the form $u = Ax^a + Bx^b + Cx^c + \&c.$, may be transformed to $\frac{du}{dx} = aAx^{a-1} + bBx^{b-1} + cCx^{c-1} + \&c.$

It may be said that other explanations, by different authors, perform the same transformations. They, however, first consider n to have some finite value which causes both u and x to change their values, but finally they assume $n = 0$, so that both u and x change back again to what they were at first; making two unnecessary changes as it seems to me.

My explanation supposes the values of u and x to remain the same during transformation and not to undergo any change at all; so that differentiation is simply Algebraic transformation; thus showing the connection between Algebra and the Calculus.

SOLUTION OF AN INDETERMINATE PROBLEM.

BY DAVID S. HART, A. M., M. D., STONINGTON, CONN.

[AT the conclusion of Dr. Matteson's solution, in Vol. II, page 49 of the ANALYST, we alluded to a request that had been made by Prof. Henkle, soon after the publication of Prof. Perkins' solution, that we publish the solution of a similar question that had been made by Dr. Hart some 20 years before, and promised to do so at some future time. As we have recently been reminded by Dr. Hart that his solution had not yet appeared in the ANALYST, we subjoin the solution alluded to, with the following prefatory remarks by Dr. Hart.—Ed.]

"This problem was I believe first given by Dr. J. R. Young in his Algebra, published a little more than 60 years ago. The answers he gives are small, but erroneous. Samuel Ward, in his American Edition of Young's Algebra, was the first to detect the error. It is probable that he went no farther than to satisfy himself that the answers must be very large numbers.

"It is now 26 years since I completed a general solution of this problem without conditions annexed. The condition that the squares shall be in arithmetical progression was solved by Prof. Kirkwood in Stoddard and Henkle's University Algebra, but his solution is a particular one involving very large numbers. Prof. Perkins has a general solution in the ANALYST, but it is limited to the case above mentioned. He finds the same numbers which I had already found, unknown to him, 25 years before.